

Dispersion compensation in atom interferometry by a Sagnac phase

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We reanalyzed our atom interferometer measurement of the electric polarizability of lithium now accounting for the Sagnac effect due to Earth rotation. The resulting correction to the polarizability is very small but the visibility as a function of the applied phase shift is now better explained. The fact that the Sagnac and polarizability phase shifts are both proportional to v^{-1} , where v is the atom velocity, suggests that a phase shift of the Sagnac type could be used as a counterphase to compensate the electric polarizability phase shift. This exact compensation opens the way to higher accuracy measurements of atomic polarizabilities and we discuss how this can be practically done and the final limitations of the proposed technique.

I. INTRODUCTION

We have measured the electric polarizability of lithium atoms [1, 2] by atom interferometry, with an experiment very similar to the one of C. R. Ekstrom *et al.* on sodium [3]. The measured phase shift as a function of the applied voltage was well fitted by a theoretical analysis in which the Sagnac phase shift due to the Earth rotation was neglected. As these two phase shifts are both dependent on the atom velocity, this omission has an effect on the predicted fringe visibility. In the present paper, we reanalyze our data with the Sagnac phase taken into account. The correction to the polarizability value is very small but the new fit corrects substantially the best fit width of the velocity distribution of the atomic beam.

This result would have only a minor interest if it does not suggest a way of improving polarizability measurements by atom interferometry. As discussed by T. D. Roberts *et al.* [4], the main factor limiting the accuracy of this measurement comes from the dependence of the polarizability phase shift $\Delta\phi_{pol}$ with the atom velocity v ($\Delta\phi_{pol} \propto v^{-1}$) and this paper developed an experiment in which the polarizability phase shift was compensated by a velocity dependent counterphase. In this way, it was possible to increase the maximum observable phase shift, with the goal of improving the measurement accuracy. The counterphase used in reference [4] was not exactly proportional to v^{-1} and this defect has probably limited the performance of this technique. We neverthe-

less think that the idea of a velocity dependent counterphase is excellent and we propose to use the Sagnac phase shift [5, 6, 7, 8, 9] as a counterphase. As this phase shift is exactly proportional to v^{-1} , it should provide an ideal compensation. To produce a Sagnac phase shift, it is possible to rotate the whole atom interferometer, as done by Lenef *et al.* [10], but more convenient techniques can be used. For example, when the atom wave is diffracted by lasers, moving only two small mirrors in opposite directions would have the same effect. In the case of Raman diffraction, a phase shift proportional to v^{-1} can be created by varying the Raman frequency shift and such a phase shift has already been used in gyroscopes based on atom interferometry [11, 12, 13]. We thus think that we can develop a very accurate counterphase which will be useful for high-accuracy measurements of atom electric polarizability.

II. ELECTRIC POLARIZABILITY MEASUREMENT BY ATOM INTERFEROMETRY

The principle of an electric polarizability measurement by atom interferometry [3] is to use a Mach-Zehnder atom interferometer in which the atomic beams can be separated by a septum and to apply an electric field on only one of the two atomic beams (see figure 1). For a given voltage U applied on the capacitor creating the field, the polarizability phase shift $\Delta\phi_{pol}(U)$ of the atomic wave is given by:

$$\Delta\phi_{pol}(U) = \frac{2\pi\epsilon_0\alpha}{\hbar v} \int E^2(z)dz \quad (1)$$

Here v is the atom velocity $v = \hbar k/m$ (for more details, see references [1, 2]). An important point here is that

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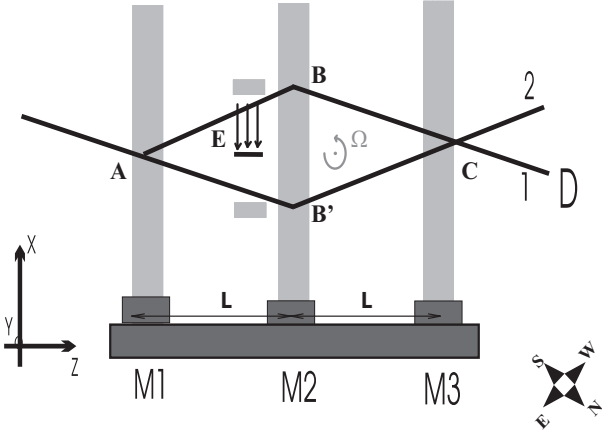


FIG. 1: Our measurement of an electric polarizability with a Mach-Zehnder atom interferometer: a collimated supersonic lithium beam is diffracted, in the Bragg regime, by three laser standing waves produced by reflecting three laser beams on the mirrors M_1 , M_2 and M_3 . The output beam labelled 1 is selected by a slit and detected by surface ionization on a hot wire detector D . A capacitor with a thin electrode is inserted between the two interfering atomic beams, just before the second laser standing wave, where the distance between the two atomic beams is largest. The electric field E is applied on the upper path of the interferometer. The orientation of the experiment is roughly represented as well as the sense of the Earth angular velocity component Ω_y .

the polarizability phase shift $\Delta\phi_{pol}(U)$ is proportional to v^{-1} and we will write

$$\Delta\phi_{pol}(U) = \Delta\phi_{pol,m}(U)u/v \quad (2)$$

where $\Delta\phi_{pol,m}$ is the phase shift for the atomic beam mean velocity u defined below. We may note that the proportionality $\Delta\phi_{pol}(U) \propto v^{-1}$ is the result of a first order perturbation calculation, valid if the ratio of the polarizability energy term to the atom kinetic energy (i.e. the ratio $2\pi\epsilon_0\alpha E^2(z)/(\hbar^2 k^2/2m)$) is considerably smaller than unity. In the experiments with thermal atoms [1, 2, 3], this ratio is typically of the order of 10^{-8} and this approximation is excellent.

Finally, the velocity dependence of the phase shift coupled to the velocity dispersion of the atomic beam induces a rapid reduction of the fringe visibility when the applied phase shift increases: as discussed in [4], this effect limits the maximum measurable phase shift and ultimately the precision of the polarizability measurement.

III. SAGNAC EFFECT

Atom interferometers are extremely sensitive to inertial effects and in particular to rotation of the setup, through Sagnac effect [5, 6, 7, 8, 9]. Several gyroscopes based on atom interferometry have been developed

[9, 10, 11, 12, 13] and an extremely high sensitivity has been achieved [12]. The Sagnac phase shift due to a rotation of the setup is given by:

$$\Delta\phi_{Sagnac} = 2k_G\Omega_y L^2/v \quad (3)$$

Here, k_G is the grating wave vector ($k_G = 2k_L$ in the case of laser diffraction by a laser with a wave vector k_L); Ω_y is the y -component of the angular velocity of the laboratory frame with respect to a Galilean frame (the y -axis being perpendicular to the plane of the atom trajectory, see figure 1); finally L is the distance between consecutive gratings. In our experiment, the mean value of Ω_y is due to the Earth rotation, while the seismic and laboratory vibrations induce rapid fluctuations of Ω_y . The main effect of these fluctuations is to induce a phase noise which reduces the fringe visibility [14, 15]. With a period equal to the sidereal day, $\Omega_y = 5.025 \times 10^{-5}$ rad/s at the laboratory latitude $\lambda = 43^\circ 33' 37''$, $k_L = 9.364 \times 10^6$ m $^{-1}$ and $L = 0.605$ m, the calculated Sagnac phase shift is given by:

$$\Delta\phi_{Sagnac} = 0.646 \times u/v \text{ rad} \quad (4)$$

Here u is the beam mean velocity defined by equation (5) below, and $u = 1065.7 \pm 5.8$ m/s as measured in [1, 2].

IV. REANALYSIS OF OUR EXPERIMENTAL SIGNALS

As the phase shifts $\Delta\phi_{pol}$ and $\Delta\phi_{Sagnac}$ are velocity dependent, both in v^{-1} , we must describe the velocity distribution of the atomic beam. As in references [1, 2], we will use:

$$P(v) = \frac{S_{\parallel}}{u\sqrt{\pi}} \exp \left[-((v-u)S_{\parallel}/u)^2 \right] \quad (5)$$

This equation is a simplified form of the velocity distribution of supersonic beams [16, 17], with S_{\parallel} , the parallel speed ratio, defined such that $S_{\parallel} = u/(\sigma\sqrt{2})$ where σ is the RMS about the mean velocity. In fact, $P(v)$ is not the velocity distribution of the incident beam but the velocity distribution of the atoms contributing to the fringe signal i.e. it is the product of the velocity distribution of the incident beam by the transmission of the interferometer. This transmission is a function of the atom velocity, in particular because of the use of Bragg diffraction. Noting $\Delta\phi = \Delta\phi_{pol}(U) + \Delta\phi_{Sagnac}$, the interferometer signal is given by:

$$\begin{aligned} I &= I_0 \int dv P(v) [1 + \mathcal{V}_0 \cos(\psi + \Delta\phi)] \\ &= I_0 [1 + \langle \mathcal{V} \rangle \cos(\psi + \langle \Delta\phi \rangle)] \end{aligned} \quad (6)$$

ψ is a phase shift function of the standing wave positions, which serves to observe interference fringes and which is independent of the atom velocity. Equation (6) defines the observed visibility $\langle \mathcal{V} \rangle$ and the observed phase shift $\langle \Delta \phi \rangle$. These two averages are non linear, so that:

$$\langle \Delta \phi \rangle \neq \int dv P(v) \Delta \phi(v) \quad (7)$$

As a consequence, after taking the average, the polarizability and Sagnac phase shifts are not exactly additive. To measure the effect of an applied voltage U , we make two measurements of the fringe phase, one with $U = 0$ and one with U and the difference provides our measurement of the polarizability phase shift $(\Delta \phi_{pol}(U))_{meas}$ given by:

$$(\Delta \phi_{pol}(U))_{meas} = \langle \Delta \phi_{pol}(U) + \Delta \phi_{Sagnac} \rangle - \langle \Delta \phi_{Sagnac} \rangle \quad (8)$$

As the electric field was applied on the ABC beam of the interferometer (see figure 1), one can verify that the polarizability and Sagnac phase shifts have opposite signs. We use equations (5,6,8) to fit our data with only two adjustable parameters, the parallel speed ratio S_{\parallel} and the ratio $\Delta \phi_{pol,m}(U)/U^2$. The phase shift and visibility data are treated in a single fit and we get:

$$S_{\parallel} = 7.67 \pm 0.06 \quad (9)$$

$$\frac{\Delta \phi_{pol,m}(U)}{U^2} = (1.3880 \pm 0.0010) \times 10^{-4} \text{ rad/V}^2 \quad (10)$$

In our initial fit neglecting the Sagnac effect [1, 2], the parallel speed ratio S_{\parallel} was found equal to $S_{\parallel} = 8.00 \pm 0.06$ and this overestimation can be explained: the Sagnac and polarizability phase shifts being of opposite signs, the visibility decay is delayed by the existence of the Sagnac phase shift. This delay induced the initial fit toward a velocity dispersion smaller than its actual value.

The $\Delta \phi_{pol,m}(U)/U^2$ value is only slightly modified with respect to its previous value equal to $\Delta \phi_{pol,m}(U)/U^2 = (1.3870 \pm 0.0010) \times 10^{-4} \text{ rad/V}^2$. This is not surprising because the effect is very indirect, through the non-linear character of the average defined by equation (6). Our new value of the polarizability of ^7Li is:

$$\alpha(\text{Li}) = (24.34 \pm 0.16) \times 10^{-30} \text{ m}^3 = 164.3 \pm 1.1 \text{ u.a.} \quad (11)$$

The 0.07 % correction is considerably smaller than the 0.66 % final error bar dominated by the uncertainty on the mean velocity u . Figures 2 and 3 show the agreement between the data points and their fits. On the visibility data, the improvement is noticeable, especially when the measured phase shifts are smaller than 10 rad.

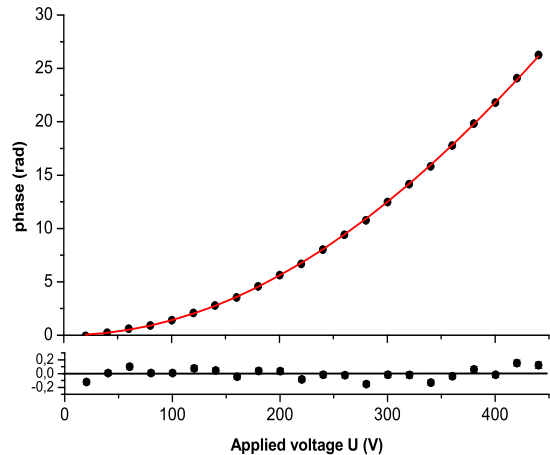


FIG. 2: (Color online) Measured phase shift $\Delta \phi_{pol,m}(U)$ in radians as a function of the applied voltage U : the best fit is represented by the full curve and the residuals are plotted in the lower graph with an expanded scale.

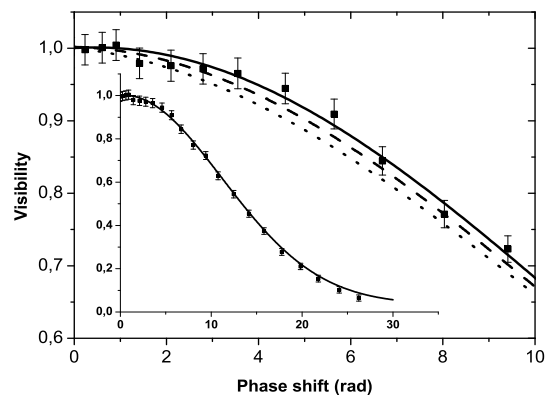


FIG. 3: The reduced fringe visibility $\langle \mathcal{V}(U) \rangle / \langle \mathcal{V}(U=0) \rangle$ is plotted as a function of the measured phase shift $\Delta \phi_{pol,m}(U)$ (the zero-field visibility is equal to $\langle \mathcal{V}(U=0) \rangle = 62\%$). The data points are plotted with their error bars and three different curves are plotted: with the correct Sagnac phase shift (full curve), without the Sagnac phase shift (dashed curve) and finally with the Sagnac phase shift taken into account with the wrong sign (dotted curve). The insert shows the new fit taking into account the Sagnac term over our complete data set.

V. THE USE OF A SAGNAC COUNTERPHASE FOR DISPERSION COMPENSATION

The fact that the polarizability phase shift is proportional to v^{-1} limits the accuracy of the measurement for three main reasons:

- the visibility decreases rapidly when the mean

phase shift increases and for sufficiently large phase shifts, the phase sensitivity is too small to be useful. T. D. Roberts *et al.* [4] have made a quantitative analysis of this effect, with an evaluation of the optimum phase shift in the case of a v^{-n} dependence of the phase shift with the velocity v .

- as shown by equations (6,8), the measured phase shift $(\Delta\phi_{pol}(U))_{meas}$ differs from the phase shift $\Delta\phi_{pol,m}(U)$ corresponding to the mean velocity u (see also Appendix B of ref. [2]). This effect is very important when the velocity distribution is broad and it forbids a very accurate determination of the polarizability.
- finally, the mean velocity u is difficult to determine with great accuracy, especially because the incident beam mean velocity is slightly modified by the velocity dependent transmission of the interferometer.

T. D. Roberts *et al.* proposed to compensate the polarizability phase shift by an engineered counterphase. To produce this counterphase, two time dependent phase shifts $\phi_{1,2}(t)$ were applied at two points separated by a distance $L_{shifters}$ along the atom path in the interferometer. The counterphase $\phi_{counter}$ is thus given by:

$$\phi_{counter} = \phi_1(t) + \phi_2(t + L_{shifters}/v) \quad (12)$$

We will not reproduce the discussion of T. D. Roberts *et al.* to which we refer the reader. However, we point out, that in addition to the velocity dependence due to the $L_{shifters}/v$ term, there is a direct velocity dependence in v^{-2} of ϕ_1 and ϕ_2 due to the fact that these phase shifts are produced by applying an electric field gradient on the atomic beams inside the interferometer. This supplementary velocity dependence, which was not discussed in reference [4], complicates the use of this counterphase, and we note that a high accuracy use of this technique remains to be demonstrated

VI. THE USE OF SAGNAC PHASE SHIFT FOR DISPERSION COMPENSATION

The observation presented in this paper suggests that one may use the Sagnac phase shift as a counterphase to compensate the polarizability phase shift. As both phase shifts are proportional to v^{-1} , the compensation should be exact and the only problem is to find a practical way of creating a large Sagnac phase shift.

If we move the mirrors of the first and/or third standing waves in opposite directions, these motions will exactly mimic a rotation of the interferometer. If the velocity of the mirrors M_1 and M_3 are respectively v_1 and v_3 , the induced phase shift is given by [18] :

$$\Delta\phi_{Sagnac} = 2k_L(v_1 - v_3)L/v \quad (13)$$

This idea seems nice, especially as piezo-actuators with capacitive displacement sensors are available, with an uncertainty on the displacement smaller than 1 nanometer. For a phase shift $\Delta\phi_{Sagnac} = 100$ radians, the needed velocities are quite large $v_1 = -v_3 = 4.7 \times 10^{-3}$ m/s, which can be sustained only during about 4×10^{-3} s if the displacement is limited to 20 micrometers. The mirrors should be moved forward and backward with a few millisecond period. This rapid motion will perturb the interferometer and we think that this technique is not practically feasible.

However, what we need is to change the positions of the nodes and anti-nodes of the laser standing waves and a two-prism arrangement as schematically represented in figure 4 can do the same effect. A displacement δz of the prism is equivalent to a change of the mirror position δx given by:

$$\delta x = \delta z \frac{1 - n \cos(i - r)}{n} = \delta z \frac{1 - n^2}{n(1 + n^2)} \quad (14)$$

where i and r are the incidence and refraction angles of the laser beam on the prism hypotenuse. The final form assumes Brewster incidence as drawn on figure 4 (i.e. $\tan i = n$ where n is the index of refraction of the prism). With fused silica ($n \approx 1.46$), the ratio $\delta x/\delta z$ is close to -0.25 and such an arrangement can replace a mirror displacement of several millimeters. It thus seems feasible to produce a Sagnac type phase shift of the order of 100 radians during a time period of the order of 1 second. Commercially available translation stages with interferometric control of the position can provide a very stable prism velocity which is needed because any velocity fluctuation will induce a phase noise.

The prism motion will induce a large Sagnac counterphase $\Delta\phi_{Sagnac}$ and the measurement will give $(\Delta\phi_{tot})_{meas} = \langle\Delta\phi_{pol}(U)\rangle + \langle\Delta\phi_{Sagnac}\rangle$. We must tune the Sagnac phase till $(\Delta\phi_{tot})_{meas}$ is very small and then its value is, with a good approximation, the value for the mean velocity u :

$$\frac{2\pi\epsilon_0\alpha}{\hbar} \int E^2(z)dz = 2k_L(v_1 - v_3)L + u(\Delta\phi_{tot})_{meas} \quad (15)$$

If the phase $\langle\Delta\phi\rangle$ is very small, a low accuracy on u is sufficient for an accurate measurement of the polarizability α . This measurement will require an accurate measurement of the quantity $2k_L(v_1 - v_3)L$. The laser wave vector k_L is easily measured with a very high accuracy and it is possible to measure the velocity v_1 and/or v_3 with a high accuracy by optical interferometry (the laser standing waves used for diffraction can be used in Michelson interferometers for this measurement). The distance L between laser standing waves may be difficult to measure with an accuracy better than about 10^{-3} , because, as shown by C.J. Bordé [19] and Ch. Antoine [20, 21], the exact value of the distance L is not exactly equal to the physical distance between the laser standing waves

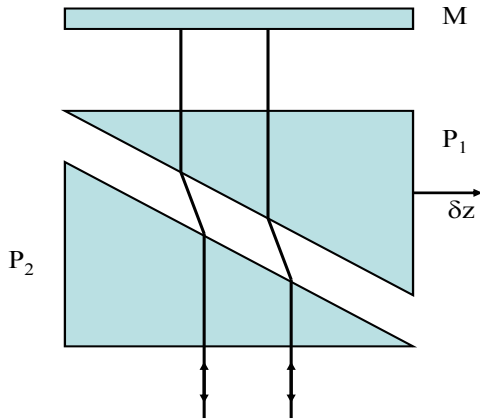


FIG. 4: (Color online) Two-prism arrangement proposed to create a Sagnac-type phase shift. The two prisms P_1 and P_2 are identical and they are introduced on the laser standing wave between the atomic beams and the mirror M . Their angle is chosen such that the laser beam can be at normal on one face and at Brewster incidence on the other face. The displacement δz of prism P_1 induces a modification δx of the one-way optical path for the laser, which is equivalent to a δx displacement of the mirror perpendicular to its surface

but its value depends on the diffraction regime and laser beam parameters.

Another technique to create a Sagnac type counterphase is to use laser Raman diffraction. This diffraction process, which is most commonly used in cold atom interferometers, has also been used by T. L. Gustavson *et al.* in their thermal atom gyrometer [11, 12]. With Raman diffraction, the phase of the interferometer signal can be modified by changing the frequency difference of the Raman laser beams. A Sagnac phase can be mimicked by applying opposite frequency offsets to the Raman beams used for the first and third diffraction events. T. L. Gustavson *et al.* [12] have shown that this phase shift is proportional to v^{-1} and they used it to compensate the Sagnac phase shift. Such an electronic compensation can be used to compensate a very large polarizability phase shift.

VII. CONCLUSION

In this paper, we have reanalyzed our measurement of the electric polarizability of lithium atom by atom inter-

ferometry and this reanalysis has taken into account the Sagnac phase shift due to Earth rotation, an effect which was ignored in our first analysis. The new value of the lithium electric polarizability differs only slightly (by 0.07 %) from our previous value, well within our 0.66 % error bar. However, the fit to the visibility data is improved and the deduced value of the parallel speed ratio S_{\parallel} is noticeably different.

The polarizability phase shift and the Sagnac phase shift have the same v^{-1} dependence with the atom velocity v and this remark suggests that the Sagnac phase shift can be used to compensate the polarizability phase shift. The idea of compensating the polarizability phase shift by a counterphase was initially developed and demonstrated by T. D. Roberts *et al.* [4] and this technique should give access to high accuracy measurements of electric polarizability. In the experiment of T. D. Roberts *et al.*, the counterphase had not exactly the same velocity dependence and this effect limits the possible accuracy. With a Sagnac phase shift as a counterphase, the compensation should be exact and the polarizability measurement will be replaced by a measurement of mirror velocity. Furthermore, if the interferometer uses Raman diffraction, the Sagnac phase can be mimicked by laser frequency offsets, which are even simpler to measure. In both cases, the size L of the interferometer plays a role and its exact value will depend on the diffraction regime. The test of these compensation schemes require substantial modifications of our atom interferometer so that they cannot be done immediately but we expect to do so in a near future.

VIII. ACKNOWLEDGEMENTS

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- [1] A. Miffre, M. Jacquy, M. Büchner, G. Tréneç and J. Vigué, Phys. Rev. A **73**, 011603(R) (2006)
 - [2] A. Miffre, M. Jacquy, M. Büchner, G. Tréneç and J. Vigué, Eur. Phys. J. D. **38**, 353 (2006)
 - [3] C. R. Ekstrom, J. Schmiedmayer, M. S. Chapman, T. D. Hammond and D. E. Pritchard, Phys. Rev. A **51**, 3883

- (1995)
- [4] T. D. Roberts, A. D. Cronin, M. V. Tiberg and D. E. Pritchard, Phys. Rev. Lett. **92**, 060405 (2004)
- [5] A. W. Overhauser and R. Collela, Phys. Rev. Lett. **33**, 1237 (1974)
- [6] R. Collela, A. W. Overhauser and S. A. Werner, Phys.

- Rev. Lett. **34**, 1472 (1975)
- [7] J. Anandan, Phys. Rev. D **15**, 1448 (1977)
 - [8] S. A. Werner, J.-L. Staudenmann and R. Collela, Phys. Rev. Lett. **42**, 1103 (1979)
 - [9] F. Riehle, Th. Kisters, A. Witte, J. Helmcke and Ch. J. Bord, Phys. Rev. Lett. **67**, 177 (1991)
 - [10] A. Lenef, T. D. Hammond, E. T. Smith, M. S. Chapman, R. A. Rubenstein and D. E. Pritchard, Phys. Rev. Lett. **78**, 760 (1997)
 - [11] T. L. Gustavson, P. Bouyer and M. A. Kasevich, Phys. Rev. Lett. **78**, 2046 (1997)
 - [12] T. L. Gustavson, A. Landragin and M. A. Kasevich, Class. Quantum Grav. **17**, 2385 (2000)
 - [13] B. Canuel, F. Leduc, D. Holleville, A. Gauguier, J. Fils, A. Virdis, A. Clairon, N. Dimarcq, Ch. J. Bordé, A. Landragin and P. Bouyer, Phys. Rev. Lett. **97**, 010402 (2006)
 - [14] A. Miffre, M. Jacquey, M. Büchner, G. Trénec and J. Vigué, Appl Phys. B **84**, 617 (2006)
 - [15] M. Jacquey, A. Miffre, M. Büchner, G. Trénec and J. Vigué, Europhysics Letters **75**, 688-694 (2006)
 - [16] H. C. W. Beijerinck, G. H. Kaashoek, J. P. M. Beijers and M. J. Verheijen, Physica **121 C**, 425 (1983)
 - [17] H. Haberland, U. Buck and M. Tolle, Rev. Sci. Instrum. **56**, 1712 (1985)
 - [18] J. Schmiedmayer, M. S. Chapman, C. R. Ekstrom, T. D. Hammond, D. A. Kokorowski, A. Lenef, R. A. Rubenstein, E. T. Smith and D. E. Pritchard in Atom interferometry edited by P. R. Berman (Academic Press, 1997), p. 1
 - [19] Ch. J. Bordé, General Relativity and Gravitation, **36**, 475 (2004)
 - [20] C. Antoine, PhD thesis, Université Paris 6, <http://tel.archives-ouvertes.fr/docs/00/04/75/84/PDF/tel-00007967.pdf>
 - [21] C. Antoine, Appl. Phys. B **84**, 585 (2006)